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## Creating Matter at the Electroweak Phase Transition\*

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I summarize recent results in which we make quantitative predictions for the baryon asymmetry of the universe in the charge transport mechanism of electroweak baryogenesis. Making favorable assumptions about the unknown quantities relevant to the problem, we find that the mechanism is marginally capable of creating a baryon asymmetry as large as that observed.

Although feasible explanations for the observed preponderance of matter over antimatter have long existed, it is only since the idea of creating it at the electroweak phase transition (EWPT) that we have the hope of testing such theories in the laboratory. Since the EWPT occurred when the universe was at a temperature near 100 GeV, any new physics it might need for producing the baryon asymmetry should be within the reach of the next generation of particle accelerators.

In fact one of the attractive features of electroweak baryogenesis is that a minimal number of new physics ingredients is needed to produce the baryon asymmetry. Already within the standard model one has baryon number violation via electroweak sphaleron interactions at high temperature. These interactions involve 9 quarks and 3 leptons, thus violating left-handed  $B + L$  by 3 units, and their rate is large up until the EWPT. Furthermore one has departure from thermal equilibrium because the EWPT is first order, proceeding by the nucleation of bubbles of the true vacuum phase (where the Higgs field gets a VEV) from the symmetric phase. Thus two of Sakharov's conditions for baryogenesis are fulfilled. The third, CP violation, is also present in the standard model, but most consider the form it takes there to be too feeble for the purposes of baryogenesis, being suppressed by factors of all the quark masses and mixing angles, and thermal damping effects. This shortcoming can be remedied by adding a second Higgs doublet, for example.

A conceptually attractive way of putting these ingredients together was proposed by Cohen, Kaplan and Nelson in 1991 [1], and elaborated more recently by Joyce, Prokopec and Turok [2]. The idea is that fermions from the symmetric phase have some probability of bouncing off the expanding walls of the bubbles of true vacuum phase. Due to CP violation in the walls, the reflection probability is different for right-handed and left-handed fermions, and so an excess of chirality initially builds up in front of the moving wall. Since sphalerons "see" only left-handed fermions, they act so as to redistribute the chirality asymmetry among all

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generations, reducing the asymmetry of the original species that was reflected most strongly. The sphalerons are thereby biased to change the net baryon number away from zero. Eventually the wall catches up to the reflected particles which are being slowed by collisions. Once the created baryons fall inside the bubble, they are safe from further destruction because the sphaleron interactions are suppressed inside the bubble, if the EWPT is strongly enough first order.

To make a quantitative estimate of the baryon production from this mechanism we have looked at the steps in some detail, attempting to replace assumptions made by previous investigators with results based on a viable model. One focus of our work was the precise nature of CP violation which, in the two-Higgs-doublet extension of the standard model, comes from the relative phase  $\theta(x)$  between the Higgs fields. Assuming that the fermion of interest couples for example to just the second of these two fields, in order to avoid flavor-changing neutral currents, it gets a complex, spatially varying mass term  $m(z) = \rho_2(z)e^{-i\theta(z)}$  in the vicinity of the bubble wall where the modulus  $\rho_2(z)$  is changing from zero outside the bubble to its VEV inside. The space-dependent mass gives rise to quantum mechanical reflection and transmission of the fermion, just as in a one-dimensional potential problem, and the phase  $\theta$  causes different-chirality particles to be reflected with different probabilities. Until now it was always assumed that  $\theta(z)$  was simply proportional to  $\rho_2(z)$ . One of our goals was to find the actual form of  $\theta(z)$  in a realistic model and to see how strongly the results depended strongly on this assumption.

More generally, we wanted to eliminate as many assumptions as possible, such as the bubble wall width, and so it behooved us (1) to construct a two-Higgs doublet model suitable for baryogenesis. Subsequently we had to (2) to find the finite-temperature form of the model; (3) solve the equations of motion for the Higgs fields  $\rho_i(z)$ ,  $\theta(z)$  near the bubble wall; (4) solve the Dirac equation in the background of the Higgs fields to find the reflection probabilities; (5) determine how the reflected fermions diffuse back into the plasma in front of the wall; and (6) to integrate these results for the baryon asymmetry. I will give only the highlights of these steps here, as the details can be found in references [3], [4].

**The Model.** To simplify the analysis we imposed the symmetry  $\phi_1 \leftrightarrow \phi_2$  on the Higgs field potential, softly broken by a dimension two operator,

$$V(\Phi_1, \Phi_2) = -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 + \tilde{\kappa} \Phi_1^\dagger \Phi_2 + \tilde{\kappa}^* \Phi_2^\dagger \Phi_1 + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + h_1 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + h_2 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + h_3 ((\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2) \quad (1)$$

Because of the symmetry, both Higgs fields will get VEV's whose moduli are equal,  $\rho_1 = \rho_2 \equiv \rho$ , and so we have reduced the problem to two fields,  $\rho(z)$  and  $\theta(z)$ . This also circumvents the complexities of a two-stage phase transition, where one field gets its VEV before the other.

The two terms responsible for CP violation at zero temperature are those with the coefficients  $\kappa$  and  $h_3$ . If either one is missing (or if their phases happen to be matched just right), all the couplings in the potential can be made real by a global field redefinition.

**Effective Potential.** We used the ring-improved finite-temperature effective potential, which is obtained from the usual one-loop finite- $T$  result by substituting the temperature-corrected masses for the zero-temperature ones. It is important to avoid the unitary gauge

in this construction, as it is known to give unreliable results near the critical temperature of the phase transition [5].

**CP-Violating Phase Profile.** The shape of the bubble wall is given by a familiar form  $\rho = \rho_c g(z)$ ,  $g(z) = \frac{1}{2}(1 + \tanh(z/\Delta))$ , where  $\rho_c$  is the Higgs field VEV inside the bubble and  $\Delta$  is the width of the wall. One finds that the equation for the shape of the phase  $\theta(z)$  is

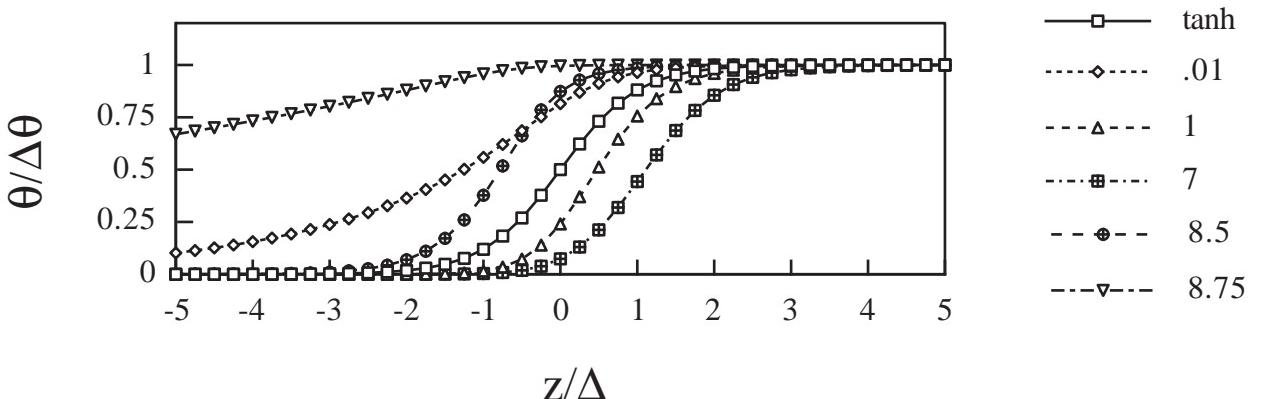
$$\Delta^2 \partial_z^2 \theta + 4\Delta(1 - g)\partial_z \theta + V(\theta) = 0, \quad (2)$$

where  $V(\theta)$  is a potential of the form

$$V(\theta) = 2|\kappa|\Delta^2 \sin(\theta - \delta) + \frac{32}{h_3} \lambda_{\text{eff}} g^2 \sin 2\theta, \quad (3)$$

and we have assumed that  $\theta$  is small so that the back-reaction of  $\theta$  on the equation for  $\rho$  could be neglected. The CP-violating phase is  $\delta$ , which is defined to be the phase of  $-\kappa$  after doing the redefinition of the Higgs fields in (1) needed to make the  $h_3$  coupling real.  $\lambda_{\text{eff}}/4$  is the effective quartic coupling in the potential for  $\rho$  after one has substituted the parametrization  $\phi_i = \rho_i e^{\pm i\theta/2}/\sqrt{2}$  into the potential (1). For most values of  $\kappa$ , the amount  $\Delta\theta$  by which  $\theta$  changes in going between the broken and symmetric phases is proportional to  $\delta$ . However it is interesting that  $V(\theta)$  has nontrivial minima even when  $\delta = 0$  if  $\kappa$  is sufficiently negative, due to the changing background field  $g(z)$ . Thus CP can be spontaneously violated during the phase transition, even though there is no sign of it in the potential at zero temperature.

Some solutions for  $\theta(z)$  are shown in figure 1, which demonstrates how they differ from the commonly used ansatz (labeled “tanh”) for different values of  $\kappa$ . The curves are distinguished by a parameter  $\zeta = -\kappa/m_h^2$ , where  $\kappa$  is by convention negative and  $m_h$  is the mass of the lightest Higgs boson, assumed to be 60 GeV. The light mass helps make the sphaleron interactions as slow as possible inside the bubble wall, so that the baryon asymmetry is not destroyed after it is created.



**Figure 1.** Solutions for the relative phase of the two Higgs fields at the bubble wall.

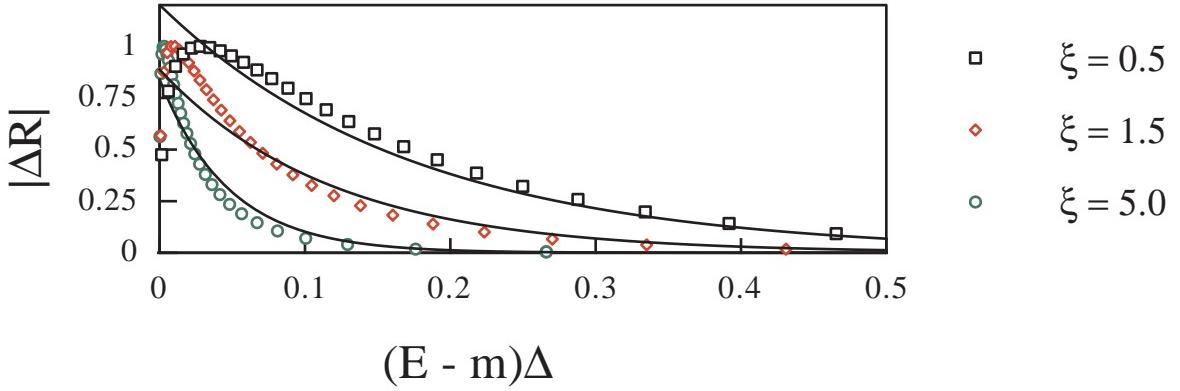
**Dirac Equation.** We solved the Dirac equation using two methods, first by directly integrating it and second using the formula of Funakubo *et al.* [6], which treats  $\theta$  as a small perturbation. We were thus able to verify the latter formula independently. The goal

was to solve for the difference in probabilities for right-handed fermions and antifermions to reflect from the wall into their opposite-chirality counterparts. The difference is called  $\Delta R \equiv R_{R \rightarrow L} - R_{\bar{R} \rightarrow \bar{L}}$ . A convenient and fairly accurate parametrization of  $\Delta R$  as a function of the fermion momentum is

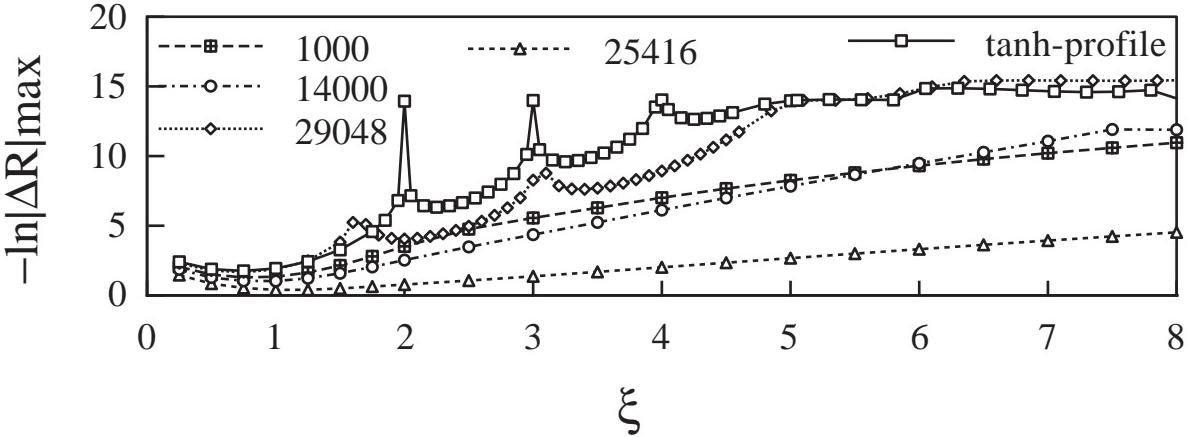
$$\Delta R(p_z) = |\Delta R|_{\max} e^{-p_z/w}, \quad p_z > m. \quad (4)$$

For  $p_z < m$ ,  $\Delta R$  vanishes because such particles do not have enough energy to penetrate into the broken phase, since they go from being massless to having a mass, and so both particles and antiparticles are completely reflected. The height and width of the exponential depends on the mass and the width of the bubble wall through their product,  $\xi = m\Delta$ .

In figure 2 the comparison between the actual functions  $\Delta R(p_z)$  and the fit (4) is shown. Although the fit is not perfect, it is only necessary to match the area under the curves because they will ultimately be integrated over  $p_z$ , combined with other functions that are rather flat on the scale over which  $\Delta R(p_z)$  is changing.



**Figure 2.** Reflection probability asymmetry for different fermion masses ( $\xi = m\Delta$ ), together with the fits (4).

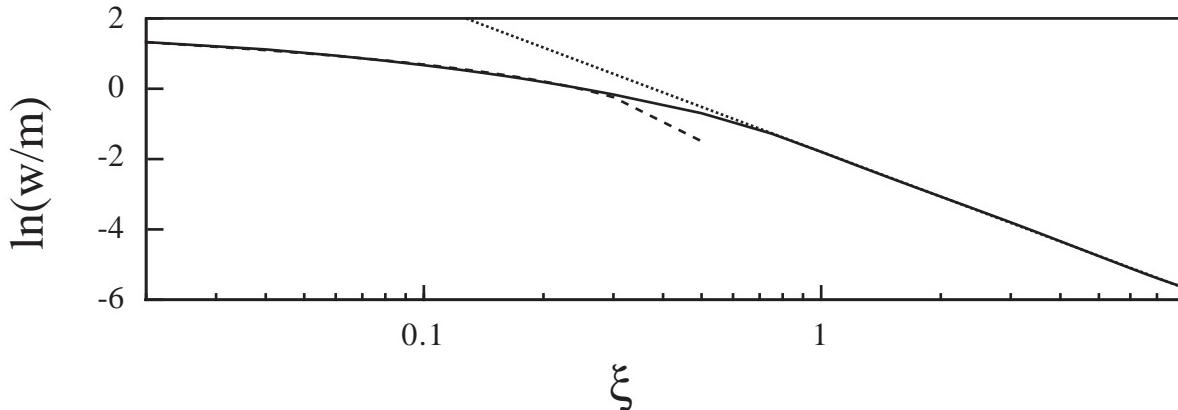


**Figure 3.** Amplitude of the reflection asymmetry as a function of fermion mass. Curves are labeled by the value of  $-\kappa$ , chosen to coincide with some of the values used in fig. 1

In figure 3 we show how the amplitude of  $\Delta R$  depends on the fermion mass parameter  $\xi =$

$m\Delta$ . The square boxes indicate the case of the ansatz  $\theta(z) \propto \rho(z)$  (called the “tanh ansatz”). The spikes are where  $|\Delta R|_{\max}$  goes through zero, which occurs near integer values of  $\xi$ . ( $\Delta R(p_z)$  does not actually vanish identically at these values of  $\xi$ ; rather the parametrization (4) breaks down, and the area under the curve  $\Delta R(p_z)$  vanishes; nevertheless its maximum value is still highly suppressed for integer values of  $\xi$ .) The other curves show the case of the real solutions for various values of  $-\kappa$ , the Higgs potential parameter. Typically these do not display the spiky behavior of the tanh ansatz, unless the corresponding  $\theta$  profiles are very close in shape to that of the tanh ansatz. One notices that the variations between the actual solutions and the ansatz are most pronounced for  $\xi > 1$ . It so happens that the actual spectrum of fermions in the standard model is  $\xi = 0.12, 0.33$  and  $11.7$  for the tau lepton, bottom quark and top quark, respectively. As will be explained below, the top quark is irrelevant for baryogenesis in this model, so for the relevant fermions, the difference between the ansatz and the actual solutions is small.

We have also examined how the width  $w(\xi)$  of the  $\Delta R$  profiles varies as a function of the fermion mass. This width is defined to be the area under the curve  $\Delta R(p_z)$  divided by the maximum value of  $\Delta R$  discussed above. The dependence is shown for typical values of the model parameters in figure 4. In contrast to the amplitude  $\Delta R_{\max}$ , we find that  $w(\xi)$  is virtually independent of the potential parameter  $\kappa$ .



**Figure 4.** Momentum-space width of the reflection asymmetry as a function of fermion mass.

An interesting but (as it turns out) inessential complication in solving the Dirac equation is that for very low momenta, the dispersion relations of the fermion are altered from their zero-temperature form,  $E = p$  in the symmetric phase or  $E = (p^2 + m^2)^{1/2}$  in the broken phase. The left- and right-handed particles get thermal contributions to their effective masses  $\omega_L$  and  $\omega_R$ , and the dispersion relations are altered to [8]

$$E = \omega_{L(R)} \pm p/3, \quad \text{symmetric phase}; \quad (5)$$

$$E = \frac{\omega_L + \omega_R}{2} \pm \sqrt{\left(\frac{\omega_L - \omega_R}{2} \pm \frac{p}{3}\right)^2 + \frac{m^2}{4}}, \quad \text{broken phase}, \quad (6)$$

in the regions of momentum space where  $|p| < \omega_i$ . The Dirac equation must be accordingly altered. However it was found that the small-momentum region makes a subdominant

contribution to the baryon asymmetry compared to the region where the usual dispersion relations prevail.

**Equilibration and Transport.** After being reflected the fermions diffuse into the symmetric phase, but before going far, interactions change their chemical composition. The most important of these are the strong sphalerons [7], the QCD analog of electroweak sphalerons. They cause chirality-changing transitions among the quarks and because they are fast, they give rise to the equilibrium condition that left-handed and right-handed baryon number are equal,  $B_L = B_R$ . On the other hand the initial condition at the wall was that the fermions had no net baryon number,  $B_L + B_R = 0$ , because it was assumed to be zero at the outset and we find that the weak sphalerons are too slow to have yet changed the net baryon number. The solution to these equations is obviously the trivial one,  $B_L = B_R = 0$ . But there are thermal corrections which make the actual solution not quite zero; instead what happens is that the asymmetries  $B_L$  and  $B_R$  get reduced by a factor  $\sim 20$  from their initial values at the wall. Of course only quarks undergo this suppression because leptons have no strong interactions. The other significant interaction is the Higgs coupling to top quarks, which we took into account in the above estimates. For the lighter fermions the Higgs coupling is too small to be important on the time-scale for fermions to diffuse in front of the wall.

The diffusion process itself has been modeled using the diffusion equation for the particle densities,

$$\frac{\partial n}{\partial t} - D \frac{\partial^2 n}{\partial x^2} = 0, \quad (7)$$

and an approximation to the Boltzmann equation for the distribution functions,

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} = \widetilde{D} \frac{\partial}{\partial \vec{p}} \cdot \left( \frac{\partial f}{\partial \vec{p}} + \beta \vec{v} f \right), \quad (8)$$

known as the Fokker-Planck equation [9]. It was argued that the latter more correctly describes the present situation because of the strong momentum dependence of the distribution functions, owing to the smallness of the momentum space width of the reflection probabilities discussed above,  $w \ll T$ . It can be shown in the one-dimensional case that the solution to the Fokker-Planck equation is only compatible with that of the diffusion equation if  $f(p)$  evaluated at the wall falls off with  $p$  like  $e^{-\beta p}$ , namely a thermal distribution. But one must remember that here  $f$  represents the asymmetry between left-handed particles and antiparticles due to the CP-violating reflections, and this asymmetry has a distribution which is far from thermal. In fact it should go like  $\Delta R(p) \sim e^{-p/w}$ , and hence the importance of the fact that  $w \ll T$ . The Fokker-Planck equation gives qualitatively different results; it predicts that the integrated asymmetry in front of the wall should be independent of the wall velocity (for a range of velocities  $v > 0.01$ ) and be cubic in the larger of  $w$  or  $m$  [9].

Even though these parametric dependences are known, an inconsistency was recently found in the numerical evaluation of integrals in ref. [9], which are needed to find the overall normalization of the integrated asymmetry in front of the wall in three dimensions. Pending the resolution of this problem, I will fall back upon the simpler diffusion equation and defer discussion of the differences between the two approaches to a future publication. Then the solution for the chiral asymmetry in front of the wall is a simple exponential,  $n(z) = v^{-1} J(0) e^{-vz/D}$ , where the initial flux  $J(0)$  at the wall can be computed from the reflection asymmetry  $\Delta R$  and the distribution functions for incident and reflected particles.

**The Baryon Asymmetry.** To compute the baryon asymmetry at a given position  $z$  in space, one must integrate the rate of baryon number violation by sphalerons from very early times (we are assuming a steady-state distribution of chirality in front of the wall) until the time the wall passes by that position. The local rate of baryon violation is proportional to the chiral asymmetry, and its time integral can be converted to the spatial integral of the asymmetry in front of the wall. The result for the ratio of baryon number to entropy of the universe is

$$\begin{aligned}\frac{n_B}{s} &= 3 \times 10^{-3} \alpha_W^4 D \frac{m^2}{T_c^2} \Delta\theta A(\xi) w(\xi) r(w/m) / v^2; \\ r(w/m) &= 1 + 2w/m + 2w^2/m^2 + K_2(m/w)e^{m/w}.\end{aligned}\quad (9)$$

not counting the suppression factor of  $\sim 20$  for contributions from the reflected quarks. Because of this factor and since the diffusion coefficient  $D$  is much smaller for quarks than for leptons, one finds that the tau lepton makes the dominant contribution [2], resulting in  $n_B/s \simeq 1 \times 10^{-12} \Delta\theta/v^2$ , which is compatible with the number determined from nucleosynthesis,  $4 - 6 \times 10^{-11}$ , but only if the wall velocity is near 0.1, the lower end of its expected range, and if CP is maximally violated. This assumes the rate per unit volume of sphaleron interactions is  $(\alpha_W T)^4$ , but it might be larger by a factor of 10, making a wall velocity of  $v = 0.3$  acceptable.

Should the CP-violating phase be too small however, there may still be ways of increasing the result; for example ref. [2] suggests that the VEV of the Higgs field coupling to the tau lepton may be larger relative to the r.m.s. VEV during the phase transition than at  $T = 0$ , so that the effective fermion mass is larger than one would infer from the zero-temperature fermion mass spectrum. Whether this can be made to happen in an actual model has not yet been demonstrated, and it would certainly involve a two-stage phase transition with its potential attendant complications. Another possibility is to alter the model so as to increase the ratio of the VEV to the temperature during the phase transition, since this would increase the ratio  $m/T$  in (10). This would also have the advantage of slowing the sphaleron interactions in the phase inside the bubbles, as one wants for preserving the baryon asymmetry.

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